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Theoretical Estimate of the Higgs Boson Mass

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It is assumed that the Higgs particle distorts space-time in its own neighborhood and generates a self-referential nonlinear field. Its almost flat space-time metric form gives a nonlinear equation of motion admitting soliton-like solutions. This in turn gives rise to a new type of wave—space-time ("mass-transmitting") interactions allowing particles to acquire mass. The curvature of the (pseudo-) Riemannian manifold of a Higgs space-time yields the mass formula $m_{W,Z}^2 = \int d^3x \sqrt{\det GR_H(\mathbf{x})} = \frac{1}{4}m_H^2$ or $m_H = 182$ GeV.

Recently, the properties of space-time at short distances and of the physical vacuum including the gravitational field (or zero-point radiation field) and fluctuations and instabilities in them have attracted great interest. Concepts relating to the metastable and false vacuum and the gravitational field (Turner and Wilczek, 1982; Guth, 1981; Linde, 1982; Kirzhnits and Linde, 1972), the properties of the vacuum as a physical medium (Lee, 1981), and fluctuations in space-time (Takano, 1961a,b; 1967) (for example, see Vigier, 1982; Namsrai, 1986) and in the Yang–Mills vacuum (Migdal, 1981) have aided in understanding very important physical problems such as phase transitions of unified theories, the very early evolution of the universe as described by the inflationary model, which can solve the monopole, horizon, and flatness problems of the standard hot big-bang cosmology, quark confinement, and so on. For the electromagnetic vacuum case, zero-point fluctuational interactions are treated on the basis that charged point-mass particles interact with a background of random classical electromagnetic zero-point radiation with definite energy spectrum. This approach is called stochastic electrodynamics (SED) (Braffort and Tzara, 1954; Braffort *et al.*, 1965; Marshall, 1965; Boyer, 1975a; Puthoff, 1987). SED yields precise quantitative agreement with

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the full QED treatment of such topics as Casimir (Lifshitz, 1955, 1956; Marshall, 1965; Boyer, 1975a) and van der Waals forces (Boyer, 1972a,b, 1973, 1975a,b), topics directly related to gravity as a zero-point-fluctuation force (Puthoff, 1989).

It seems that a universal background or radiation field, by analogy with SED, initially arose from processes in the early universe (the big bang) acting on physical objects everywhere; in particular, its different form of interaction with a matter field can be used to quantize physical systems (Parisi and Wu, 1981) and study Green functions of particles (Dineykhan *et al.*, 1991). In this connection, it should be noted that it makes no sense to say that empty space-time fluctuates or is quantized. On the contrary, physical space-time is not utterly empty: it has visible and invisible (dark) matter as background. Roughly speaking, physical space-time consists of empty space-time and backgrounds. In metric language this means that $\eta_{\mu\nu} \to G_{\mu\nu} = \eta_{\mu\nu} + \epsilon_{\mu\nu}(x)$, where $\eta_{\mu\nu}$ is the Minkowski metric of empty space-time and $\epsilon_{\mu\nu}(x)$ is some background or radiation field (medium). Our approach follows in this direction and is devoted to connecting the background field $\epsilon_{\mu\nu}(x)$ with properties of the Higgs particle, and to calculating its mass. It is well known that the beautiful unified theory of weak and electromagnetic forces (Weinberg, 1967; Salam, 1969; Glashow *et al.*, 1970) needs the Higgs mechanism for the gauge bosons to acquire mass. It seems that the Higgs particle has only mass, and no other characteristics, and it distinguishes particles from empty space. In order to give particles mass, a background field is invented which becomes locally distorted whenever a particle moves through it. The distortion—the clustering of the field around the particle—generates the particle's mass. Even if all interactions are switched off, including gravitation, its field still exists everywhere as vacuum background (viscous fluid) and fills the universe.

Thus, the Higgs particle is characterized by its mass and wave function. A question arises: How, through these quantities, can one construct a locally distorted space (background field) by which moving (interacting) particles gain mass? We assume that this distorted structure is given by the following simple perturbation form:

$$G_{\mu\nu} = \eta_{\mu\nu}(1 - \epsilon(x)) \tag{1}$$

$$\epsilon(x) = \delta \cdot m_H^{-d} |\psi(\mathbf{x}, t)|^2$$

where δ is the normalization constant, d is the dimension of space, and m_H and $\psi(\mathbf{x}, t)$ are the mass and wave function of the Higgs particle, respectively. We use units such that $\hbar = c = 1$.

Assumption (1) implies that far from the "location" of the Higgs particle, space-time is almost flat, while in its neighborhood it is slightly distorted.

For example, it looks like the form shown in Fig. 1. In this paper we study some consequences of (1).

Let us consider the case of one spatial dimension ($d = 1$, $\delta = 1$). Let the Higgs particle move in its self-referential field (1). Then, assuming that the field (1) is static, one gets (Landau and Lifshitz, 1951)

$$E = \frac{m_H}{\sqrt{1 - v^2/c^2}} \sqrt{-G_{00}} \tag{2}$$

for the Higgs particle energy. From (2) it is easy to construct the nonrelativistic Schrödinger equation

$$i \frac{\partial \psi}{\partial t} = -\frac{1}{2m_H} \frac{d^2}{dx^2} \psi + U_H(x)\psi \tag{3}$$

where

$$U_H(x) = m_H(\sqrt{-G_{00}} - 1) \approx -\beta_1 |\psi|^2 - \beta_2 |\psi|^4 - \cdots \tag{4}$$

is the Higgs potential, $\beta_1 = \frac{1}{2}$, $\beta_2 = \frac{1}{8} m_H^{-1}$. We see that equation (3) is essentially nonlinear. With the first term in (4), equation (3) is completely integrable and its solutions are solitons (Makhantov, 1990):

$$\begin{aligned} \psi_H = A \exp\{i[(1 + \tfrac{1}{2}\beta_1 m_H A^2)t + (x - vt) + \theta_0]\} \\ \times \cosh^{-1}[A(m_H\beta_1)^{1/2}(x - vt - x_0)] \end{aligned} \tag{5}$$

with the normalization constant $A = \frac{1}{2}(m_H\beta_1)^{1/2} = \frac{1}{2}(\frac{1}{2}m_H)^{1/2}$. Thus, in the stationary case the normalized metric (1) is

$$\begin{aligned} G_{00} &= -1 + \tfrac{1}{8}\cosh^{-2}(xa) \\ . \ G_{ij} &= \delta_{ij}[1 - \tfrac{1}{8}\cosh^{-2}(xa)], \qquad a = \tfrac{1}{4} m_H \end{aligned} \tag{6}$$

With the metric form (6) the Higgs potential (4) acquires the enormously deep well

$$U_H(x) = m_H(|\tanh(ax)| - 1) \tag{7}$$

with the depth $U_H^0 = -m_H$. This potential in turn gives rise to bound states [say, a particle with mass M in the Higgs field (1)] described by the stationary Schrödinger equation

$$-\frac{1}{2M} \frac{d^2}{dx^2} \psi_M - \left[E + \frac{1}{2} \frac{M}{m_H} U_H(x)\right] \psi_M = 0 \tag{8}$$

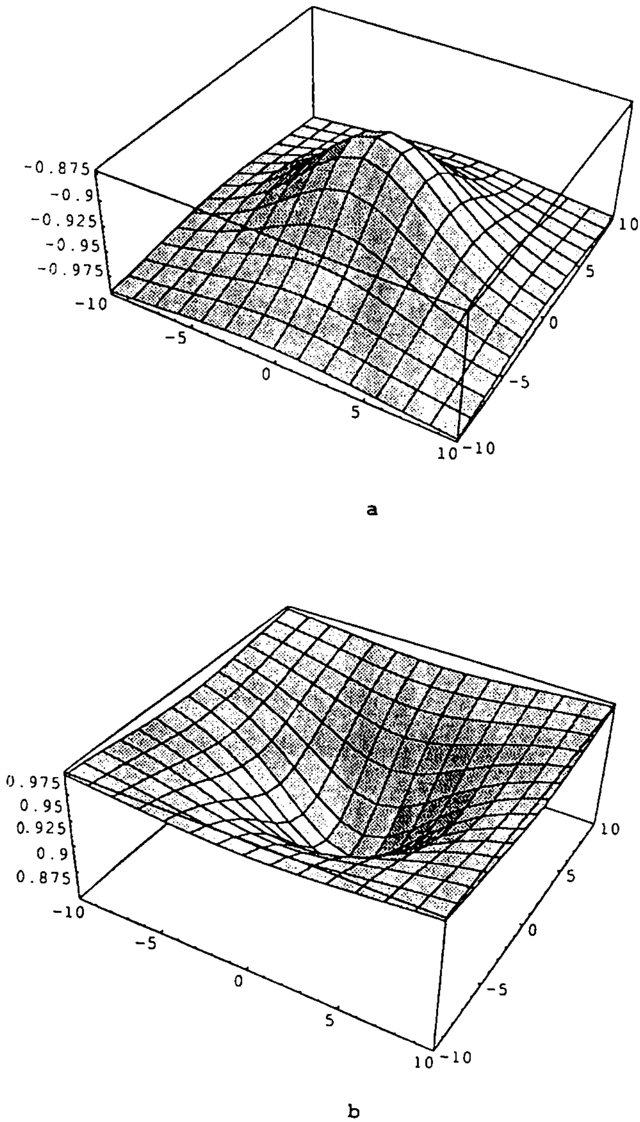

Fig. 1. The structure of space-time in the neighborhood of the Higgs boson, determined by the metric forms (a) $G_{00}(\mathbf{x})$ and (b) $G_{11}(\mathbf{x})$ in accordance with (17).

Its approximate form

$$-\frac{1}{2M}\frac{d^2}{dx^2}\psi_M - \left[E + \frac{1}{16}M\cosh^{-2}(xa)\right]\psi_M = 0 \tag{9}$$

gives the energy levels

$$E^n_{m_H,M} = -\frac{m_H^2}{32M}\left\{\frac{1}{2}\left(1 + 8\frac{M^2}{m_H^2}\right)^{1/2} - \left[\frac{1}{2} + n\right]\right\}^2 \tag{10}$$

The number of discrete levels is equal to the largest integer satisfying the inequality[2]

$$N < \tfrac{1}{2}\sqrt{1 + 8M^2 m_H^{-2}} - \tfrac{1}{2}$$

For two Higgs particles the discrete energy levels are

$$E^n_{m_H m_H} = -\tfrac{1}{32}m_H\{\tfrac{1}{2}\sqrt{1 + 16} - (n + \tfrac{1}{2})\}^2 = -\tfrac{1}{32}m_H R \tag{11}$$

where $R = 2.25$ for $n = 0$ and $R = 0.25$ for $n = 1$. This means that the Higgs particles are coupled together with a sufficiently large "force" $\sim m_H/14$.

Now we generalize the above considerations to the relativistic and four-dimensional space-time cases. The metric form (1) and equation (2) give the nonlinear Klein–Gordon equation

$$(\Box - m^2)\psi + \beta|\psi|^2\psi = 0 \tag{12}$$

where

$$\Box = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial \mathbf{x}^2}, \qquad \beta = \delta m_H^{-1} \tag{13}$$

It turns out that equation (12) admits the exact solution

$$\psi(\dot{\mathbf{x}}, t) = \exp\{-i[\omega t - \mathbf{p}\mathbf{x} + \theta_0]\}\, c(1 + \lambda x^2)^{-1} \tag{14}$$

when

$$\omega = \sqrt{m^2 + \mathbf{p}^2}, \qquad \omega t - \mathbf{p}\mathbf{x} = \sigma^i U_i = 0 \tag{15}$$

where

$$c = (8\lambda/\beta)^{1/2}, \qquad x^2 = \mathbf{r}^2 - x_0^2$$

[2] Particles with mass $M < m_H/\sqrt{8}$ do not form bound states with Higgs bosons. All observable particles, except for gauge bosons, have mass less than ~ 10 GeV and therefore they cannot acquire mass due to the scheme (1). Quarks and leptons acquire mass at the second-stage interaction mechanism (the Yukawa coupling to the Higgs bosons).

The last condition (15) means that the Higgs particle moves in a refractive medium with refractive index $\rho > 1$. In this case, the two four-vectors

$$\sigma^i = \left\{\frac{\nu}{c}, \frac{\nu}{w}\mathbf{n}\right\} \tag{16a}$$

$$U_i = \left\{\frac{c}{\sqrt{1 - u^2/c^2}}, \frac{\mathbf{u}}{\sqrt{1 - u^2/c^2}}\right\} \tag{16b}$$

are orthogonal to each other, i.e., the phase of the wave in (14) satisfies the condition (15) identically. Here σ^i and U_j are the wave number and the ray velocity vectors, respectively. Indeed, if we assume $\mathbf{u} = d\mathbf{r}/dt$ is the three-ray velocity of the Higgs particle, then the invariant phase F of the wave (14) is

$$F = \omega t - \mathbf{pr} = Et - E\left(\frac{E}{p}\right)^{-1}\left(\frac{\mathbf{p}}{p}\right)\mathbf{r}$$

$$= \nu t - \frac{\nu}{w}\mathbf{nr} = 0, \qquad E = \sqrt{\mathbf{p}^2 + m^2}$$

or

$$\nu - \frac{\nu}{w}\mathbf{n}\frac{d\mathbf{r}}{dt} = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}\left(\nu - \frac{\nu}{w}\mathbf{nu}\right) = \sigma^i U_i$$

Here σ^i and U_i are defined by (16), where $\mathbf{n}$ denotes a unit three-vector in the direction of the wave normal, ν is the frequency ($\omega = E = \hbar\nu$), $w = (p^2 + m^2)^{1/2}/p$ is the phase velocity, $\mathbf{u} = u\mathbf{e}$, $\mathbf{e}$ is a unit three vector in the direction of the ray, and u is the magnitude of the ray velocity. In the rest system O' of the refractive medium where $\mathbf{e}' = \mathbf{n}'$ and $w' = u' = c/\rho$, we have

$$(\sigma^i)' = \left\{\frac{\nu'}{c}, \frac{\nu'}{c}\rho\mathbf{n}'\right\}$$

$$U_i' = \left\{\frac{\rho c}{\sqrt{\rho^2 - 1}}, \frac{c\mathbf{n}'}{\sqrt{\rho^2 - 1}}\right\}$$

From this we immediately conclude that

$$\sigma^i U_i = (\sigma')^i U_i' = 0$$

Furthermore, we are interested in the stationary metric form determined by the relation (14). The normalization condition

$$\int d^3x \, |\psi(\mathbf{x}, 0)|^2 = 1$$

and the requirement of equality of the metric $G_{\mu\nu}$ at the initial point $\mathbf{x} = 0$ of the coordinate system for cases of one and three spatial dimensions give the unique choice of the parameters δ and λ in (14). Thus, expression (6) acquires the form (Fig. 1)

$$G_{00}(\mathbf{x}) = -1 + \tfrac{1}{8}(1 + \tfrac{1}{64}\mathbf{x}^2 m_H^2)^{-2} \tag{17}$$
$$G_{ij} = 1 - \tfrac{1}{8}(1 + \tfrac{1}{64}\mathbf{x}^2 m_H^2)^{-2}$$

and the Higgs potential reads

$$U_H(\mathbf{x}) = -\frac{m_H}{16}\left(1 + \frac{1}{64}\,\mathbf{x}^2 m_H^2\right)^{-2} \tag{18}$$

Now we would like to answer the following question: what is the nature of the Higgs particle? We use the standard method. We know that the form of the potential (say, the Coulomb or Yukawa law) is related to the force-transmitting quanta, the particle propagator [$1/\mathbf{p}^2$ for the photon, $(\mathbf{p}^2 + m^2)^{-1}$ for the scalar particle with mass m], in the static limit by the Fourier transform. Thus, we assume that for a particle with mass μ near the Higgs particle with potential (18) there is a "mass-transmitting" force between them and the propagator of the "mass-transmitting" quanta can be defined as

$$U_{\mu m_H}(\mathbf{r}) = \frac{g^2}{(2\pi)^3}\int d^3p \exp(i\mathbf{pr}) D_H(\mathbf{p}^2) \tag{19}$$

where

$$D_H(\mathbf{p}^2) = \frac{1}{m_H^2}\exp\left[-8\left(\frac{\mathbf{p}^2}{m_H^2}\right)^{1/2}\right]$$

and (20)

$$g = 4\pi\left(\frac{2\mu}{m_H}\right)^{1/2}$$

Furthermore, in accordance with standard quantum field theory methods, we can construct "mass-transmitting" quanta—the Higgs-particle Lagrangian, the equation of motion, and its propagator (20) in the four-dimensional space-time:

$$L_H = \frac{1}{2}\int d^4x\, \Phi_H(x) K(\Box, m_H^2)\Phi_H(x) \tag{21}$$

$$K(\Box, m_H^2)\Phi_H(x) = 0$$

$$D_H(p^2) = \frac{1}{m_H^2}\exp\left[-8\left(-\frac{p^2}{m_H^2}\right)^{1/2}\right]$$

$$p^2 = p_0^2 - \mathbf{p}^2$$

where

$$K(\Box, m_H^2) = m_H^2 \exp\left[8\left(-\frac{\Box}{m_H^2}\right)^{1/2}\right] \tag{22}$$

is the square-root nonlocal pseudodifferential operator, the action of which is given by the Fourier transform. A representation acts on a function $\Phi(\mathbf{x}, t)$ as follows:

$$\begin{aligned} K(\Box, m_H^2)\Phi(x) &= m_H^2 \cosh\left(8\left(-\frac{\Box}{m_H^2}\right)^{1/2}\right)\Phi(x) \\ &\quad + m_H^3(\Box)\sinh\left[8\left(-\frac{\Box}{m_H^2}\right)\right]\frac{1}{4\pi^2}\int d^4y\, \tilde{\Phi}(y)\frac{1}{|x-y|} \end{aligned} \tag{23}$$

The kernel function in (23) has a singularity on the diagonal $x = y$ and is a smooth function off the diagonal. The singularity on the diagonal is characteristic of pseudodifferential operators (Smith, 1993). Such a type of operator (23) appears in applications of the Bethe–Salpeter equation to bound states of quarks in the general problem of binding in very strong fields and also in string theory, and therefore Pauli's square-root operator is especially relevant to modern particle theory [for details see Smith (1993)].

We see that the "mass-transmitting" quantum, the Higgs particle, is essentially nonlocal and spreads out over the whole space; its propagator has no pole in momentum space. It is perhaps difficult to detect the Higgs particle. Thus, the assumption that the physical characteristic of the Higgs particle is its mass and that it is spread out over space, interacting with other particles to give them mass through the wave-space properties (1), is crucial in our approach. Above we have considered its wave properties and bound states; now we look at its space-time structure. We have seen that space-time in the neighborhood of Higgs particles is distorted in accordance with (1), (6), and

(17), and if other particles are near them, there is a force between them due to the curvature of the pseudo-Riemannian manifold. Let a scalar particle with mass M move in the Higgs space-time (1). Then, its Lagrangian function takes the form (Fulling, 1991)

$$L_\varphi = \tfrac{1}{2}(G^{\nu\mu}\nabla_\mu\varphi\nabla_\nu\varphi - M^2\varphi^2 - \xi R_H\varphi^2) \tag{24}$$

where $\nabla_\mu\varphi$ is the covariant derivative of φ, which reduces to the literal partial derivative of φ in our case. The Euler–Lagrange equation is

$$\Box\varphi + (M^2 + \xi R_H)\varphi = 0 \tag{25}$$

where

$$\Box\varphi = \frac{1}{\sqrt{-G}}\,\partial_\mu[G^{\mu\nu}\sqrt{-G}\partial_\nu\varphi]$$

Here we are not interested in a parameter ξ in (24) which arises from the conformal invariance [$\xi = (N-2)/4(N-1)$, in our case $\xi = 1/6$] and most likely concerns a renormalized wave function of φ. From (25) we see that even a massless particle acquires a mass value

$$M^2_{\text{acquir}} = \langle R_H\rangle_{\text{averaged}} \tag{26}$$

in the Higgs field. Here averaging of curvature is carried out over space. Let the initial mass of a particle be zero and the field φ be gauge boson. Then, by definition

$$M^2_{Z,W} = \int d^3x\,\sqrt{G}R_H(x) \tag{27}$$

where $G = |\det G_{i,j}(\mathbf{x})|$ in (17). The curvature scalar calculated by means of (17) is

$$\begin{aligned} R &= G^{ik}R_{ik} \\ &= G^{11}\left(4\,\frac{\partial\Gamma^3_{11}}{\partial x^3} - 4\,\frac{\partial\Gamma^2_{12}}{\partial x^1} + 4\,\frac{\partial\Gamma^2_{11}}{\partial x^2} - 2\Gamma^1_{11}\Gamma^1_{11} - 2\Gamma^2_{22}\Gamma^2_{22} - 2\Gamma^3_{33}\Gamma^3_{33}\right)\end{aligned}$$

where $G^{11} = G^{22} = G^{33}$ and

$$\Gamma^i_{jk} = \frac{1}{2}\,G^{in}\left(\frac{\partial G_{nj}}{\partial x^k} + \frac{\partial G_{nk}}{\partial x^j} - \frac{\partial G_{jk}}{\partial x^n}\right)$$

With the explicit form (17) all quantities are calculated in the standard way and the equality (27) is reduced to the mass formula

$$M^2_{Z,W} = \beta M^2_H \tag{28}$$

where

$$\beta = \frac{3\pi}{16}\int dy\, y^2\left(1 + \frac{1}{64}y^2\right)^{-3} \left[\left(1 + \frac{1}{64}y^2\right)^2 - \frac{1}{8}\right]^{-3/2}\left[\frac{1}{8} + \left(1 + \frac{1}{64}y^2\right)^2\left(\frac{1}{64}y^2 - 1\right)\right]$$

Numerical calculations give $m_H^2 = 4M_{Z,W}^2$. Taking $M_Z = 91.17$ GeV for the Z-boson, one gets

$$m_H = 182.5 \text{ GeV} \tag{29}$$

Indirect bounds on the Higgs boson mass ($m_H < 250$ GeV) have been obtained in Ellis *et al.* (1993).

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